Period 1, Oct 23, 2024

$\sqrt{x^2} = |x| \quad x=-3 \quad \sqrt{(-3)^2} = \sqrt{9} = 3$

19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$

20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$

21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$

22. $\frac{d}{dx}[\text{arccot } u] = \frac{-u'}{1+u^2}$

23. $\frac{d}{dx}[\text{arcsec } u] = \frac{u'}{|u|\sqrt{u^2-1}}$

24. $\frac{d}{dx}[\text{arccsc } u] = \frac{-u'}{|u|\sqrt{u^2-1}}$

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e) $f(x) = x\arcsin(1-x^2)$

$u = 1-x^2$

$u' = -2x$

$F'(x) = 1 \cdot \arcsin(1-x^2) + x \cdot \frac{u'}{\sqrt{1-u^2}} = \arcsin(1-x^2) + x \cdot \frac{-2x}{\sqrt{1-(1-x^2)^2}}$

$F'(x) = \arcsin(1-x^2) - \frac{2x^2}{\sqrt{1-(1-2x^2+x^4)}} = \arcsin(1-x^2) - \frac{2x^2}{\sqrt{\cancel{1}-\cancel{1}+2x^2-x^4}}$

$|x|\sqrt{2-x^2} = \sqrt{x^2(2-x^2)}$

$F'(x) = \arcsin(1-x^2) - \frac{2x^2}{|x|\sqrt{2-x^2}}$

d) $f(x) = x^2 \arctan x$

$F'(x) = 2x\arctan x + x^2 \cdot \frac{1}{1+x^2}$

$2x\arctan x + \frac{x^2}{1+x^2}$

$\frac{3+1}{3} = \frac{4}{3} \neq \frac{\cancel{3}+1}{\cancel{3}} = 1$

c) $f(x) = \tan^{-1}(5x)$

$u = 5x$

$u' = 5$

$F'(x) = \frac{u'}{1+u^2}$

$F'(x) = \frac{5}{1+(5x)^2} = \frac{5}{1+25x^2}$

b) $y\sqrt{x} - x\sqrt{y} = 12$ at $(9, 16)$

$$y \cdot x^{\frac{1}{2}} - x \cdot y^{\frac{1}{2}} = 12$$

$$\frac{dy}{dx} \cdot x^{\frac{1}{2}} + y \cdot \frac{1}{2\sqrt{x}} - \left[1 \cdot y^{\frac{1}{2}} + x \cdot \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx}\right] = 0$$

$$\frac{dy}{dx} \cdot \sqrt{9} + 16 \cdot \frac{1}{2\sqrt{9}} - \left[\sqrt{16} + 9 \cdot \frac{1}{2\sqrt{16}} \cdot \frac{dy}{dx}\right] = 0$$

$$3\frac{dy}{dx} + \frac{16}{6} - 4 - \frac{9}{8}\frac{dy}{dx} = 0$$

Solve For $\frac{dy}{dx}$

$\frac{32}{45} = \frac{dy}{dx} \Rightarrow$ normal slope $= \frac{-45}{32}$ Line $y - 16 = \frac{-45}{32}(x - 9)$

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2. Find the derivative of the inverse function at the indicated point.

a) If $f(x) = x^3 + 7x + 9$, find $(f^{-1})'(1)$ b) If $f(4)$

$$(F^{-1})'(1) = \frac{1}{F'(F^{-1}(1))} = \frac{1}{F'(-1)} = \boxed{\frac{1}{10}}$$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$F'(x) = 3x^2 + 7$

$F'(-1) = 3(-1)^2 + 7 = 10$

$F^{-1}(1) = a = -1$

$F(a) = 1$

$1 = a^3 + 7a + 9$

$a = -1$

$(-1)^3 + 7(-1) + 9$

$-1 - 7 + 9 = 1$

b) $y = (\sin^{-1} x)^2$ $\Rightarrow y = u^2$

$u = \sin^{-1} x$

$\frac{dy}{du} = 2u$

$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$

$\cancel{\frac{du}{dx}} \cdot \frac{dy}{\cancel{du}} = \frac{dy}{dx}$

$\frac{1}{\sqrt{1-x^2}} \cdot 2u = \frac{dy}{dx}$

$\frac{1}{\sqrt{1-x^2}} \cdot 2(\sin^{-1} x) = \frac{dy}{dx}$

$y = \sin^{-1} x$

$\sin y = x$

$\cos y \frac{dy}{dx} = 1 \Rightarrow \sqrt{1-x^2}\frac{dy}{dx} = 1$

$\sin^2 y + \cos^2 y = 1$

$\boxed{\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}}$

$(x)^2 + \cos^2 y = 1$

$\cos^2 y = 1 - x^2$

$\cos y = \sqrt{1-x^2}$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

$\sin(\arctan \frac{3}{4})$

$\sin L = ? = \frac{3}{5}$

L must be in Quad #1

$\arctan \frac{3}{4} = L$

$\tan L = \frac{3}{4}$

$\tan^2 L + 1 = \sec^2 L$

$\left(\frac{3}{4}\right)^2 + 1 = \sec^2 L$

$\frac{9}{16} + \frac{16}{16} = \sqrt{\frac{25}{16}} = \sqrt{\sec^2 L}$

Quad #1 $\Rightarrow + \frac{5}{4} = \frac{1}{\cos L}$

$\cos L = \frac{4}{5}$

$\sin L$

$\sin^2 L + \cos^2 L = 1$

$\sin^2 L + \left(\frac{4}{5}\right)^2 = 1$

$\sin^2 L = 1 - \frac{16}{25} = \frac{25}{25} - \frac{16}{25} = \frac{9}{25}$

$\sin L = \sqrt{\frac{9}{25}} = \frac{3}{5}$

$\cos\left(\arcsin\frac{5}{13}\right)$ $\qquad$ $\arcsin\frac{5}{13} = z$